Int. J. Heat Mass Transfer. Vol. 18, pp. 329–331. Pergamon Press 1975. Printed in Great Britain

SHORTER COMMUNICATIONS

THERMAL RESPONSE CHARACTERISTICS OF UNSTEADY STAGNATION POINT FLOWS: A NEW APPROACH

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(*Received* 4 *May* 1973 *and in revised form* 21 *January* 1974)

NOMENCLATURE

$a_2(\tau)$, wall shear stress function;
$g(\tau)$, freestream velocity time dependence;
Pr, Prandtl number;
qw, heat flux at wall;
qw, Qs, quasi-steady wall heat flux;
T, temperature;
t, time;
U, velocity at edge of boundary layer;
u, velocity component in x-direction;
v, velocity component in y-direction;
x, coordinate along surface;
y, coordinate normal to surface.

Greek symbols

$\Gamma(n)$, complete gamma function;
η, dimensionless y coordinate;
θ, dimensionless temperature;
$\theta_1(\tau)$, wall heat flux function;
ϕ, dimensionless stream function;
ν, kinematic viscosity;
τ, dimensionless time;
τw, wall shear stress;
$\tau w, Qs$, quasi-steady wall shear;
ω, dimensionless frequency.

Subscripts

Qs, quasi-steady;
w, wall;
∞, freestream condition;
$'$, denotes differentiation with respect to η;
$\dot{}$, denotes differentiation with respect to τ.

1. INTRODUCTION

THIS communication reports on the extension of a method developed by the author for the study of transient stagnation point flows [1] to the study of the thermal response of these flows. It is shown that the thermal boundary-layer equation is reducible to a first order ordinary differential equation which governs the transient wall heat flux for all time and yields results for highly unsteady conditions.

2. GOVERNING EQUATIONS

The thermal response of a two-dimensional, unsteady, laminar, incompressible, constant property boundary layer in the vicinity of a front stagnation point is considered. The freestream velocity is initially steady and at some instant it begins to vary in magnitude as specified by the function $g(t)$ in equation (1).

$$U(x,t) = axg(t) \text{ for } t \geqslant 0 \quad \text{and} \quad g(t) = 1 \text{ for } t < 0. \tag{1}$$

The temperature of the fluid freestream, T_∞, and the surface, T_w, are assumed to be uniform and constant for all time. The solution to this problem before the initiation of the transient freestream behavior is the well known solution for the flow and heat transfer in Hiemenz flow.

To facilitate analysis the momentum boundary-layer equations are transformed by a stream function defined by

$$u = \frac{\partial\Psi}{\partial y}, \qquad v = -\frac{\partial\Psi}{\partial x} \tag{2}$$

and by

$$\begin{gathered}\tau = at, \qquad \eta = y(ag/\nu)^{1/2},\\ \Psi(x,y,t) = x(ag\nu)^{1/2}\phi(\tau,\eta).\end{gathered} \tag{3}$$

The temperature field is transformed by

$$\theta(\tau,\eta) = \frac{T-T_\infty}{T_w - T_\infty}. \tag{4}$$

Under these transformations the flow equations take the form

$$\phi''' + \left(\phi - \frac{\eta}{2}\frac{\dot{g}}{g^2}\right)\phi'' = \frac{\dot{g}}{g^2}(\phi'-1) + \frac{\dot{\phi}'}{g} + (\phi')^2 - 1 \tag{5}$$

with

$$\phi'(\tau,0) = \phi(\tau,0) = 0 \tag{6}$$

$$\phi'(\tau,\infty) = 1 \tag{7}$$

$$\phi'(\tau<0,\eta) = \text{steady Hiemenz flow velocity profile} \tag{8}$$

and the energy equation is transformed to

$$\theta'' + Pr\left(\phi - \frac{\eta}{2}\frac{\dot{g}}{g^2}\right)\theta' = \frac{Pr}{g}\dot{\theta} \tag{9}$$

with

$$\theta(\tau,0) = 1, \qquad \theta(\tau,\infty) = 0 \tag{10}$$

and

$$\theta(\tau<0,\eta) = \text{steady temperature profiles for Hiemenz flow.} \tag{11}$$

In the preceding, the primes denote differentiation with respect to η and the dot with respect to τ and it has been assumed that $g(\tau) \neq 0$.

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3. METHOD OF SOLUTION

The method of asymptotic integration is used to integrate equations (5) and (9). Meksyn [2] has employed this technique to analyze the steady state equations. To the author's knowledge, this method has not been used for the transient equations. The flow equations were analyzed in reference [1]. The same method is now employed to analyze the transient energy equation.

In brief series solutions of the form

$$\phi(\tau,\eta) = \sum_{n=2}^{\infty} a_n(\tau)\frac{\eta^n}{n!} \tag{12}$$

$$\theta(\tau,\eta) = \sum_{n=0}^{\infty} \frac{\theta_n(\tau)\eta^n}{n!} \tag{13}$$

are assumed for equations (5) and (9). The coefficients $a_2(\tau)$ and $\theta_1(\tau)$ are unknowns and their solutions are dictated by the boundary conditions, [equations (7) and (10)], at the outer edge of the viscous and thermal layers. Physically $a_2(\tau)$ and $\theta_1(\tau)$ are proportional to the instantaneous wall shear and heat flux. Employing the method of asymptotic integration yields equations (14) and (15) which are to be solved for $a_2(\tau)$ and $\theta_1(\tau)$.

preceding phenomena is the result of acoustic streaming near the wall. For the wall shear and heat flux the maximum unsteadiness occurs while the freestream velocity is decelerating. In addition, the maximum unsteadiness of the heat flux increases with increasing Prandtl number.

Figure 1 shows the phase shifts θ of the maximums in wall shear and heat flux relative to the maximums in freestream velocity for $A = 0{\cdot}1$ and various frequencies ω. The maximum shear leads the maximum velocity and the phase advance increases with frequency and approaches the asymptotic limit of 45° predicted by Lighthill [3].

These results for shear phase advance are in excellent agreement with Ishigaki [4]. The maximum wall heat flux lags behind the maximums in velocity and this phase lag increases with frequency. It is seen that this phase lag increases beyond the asymptotic limit of 90° found by Lighthill using a first order perturbation analysis.

Figure 2 shows results for a larger amplitude case, $A = 0{\cdot}3$ and $\omega = 5{\cdot}6$. Both the wall heat flux and the shear undergo large departures from the quasi-steady solutions. Furthermore, the responses are no longer sinusoidal. The heat flux exhibits the greatest departure from sinusoidal behavior with a "spike-like" peak occurring at the point

$$\dot{a}_2\left[\frac{\Gamma(4/3)7{\cdot}6^{4/3}a_3}{270\,a_2^{4/3}}-\frac{9}{2}\right]\frac{1}{ga_2} = -16+5\Gamma(1/3)6^{1/3}a_2^{2/3}+\frac{55}{18}\Gamma(2/3)a_3\left(\frac{6}{a_2}\right)^{2/3}$$
$$-\frac{15}{8}\left(\frac{a_3}{a_2}\right)^2+\frac{27}{4}\frac{\dot{g}}{g^2}+\frac{\Gamma(4/3)}{27}\left(\frac{6}{a_2}\right)^{4/3}\left[\frac{7}{40}\frac{\dot{g}}{g^2}a_3+\frac{7}{5}\frac{(2\dot{g}^2-g\ddot{g})}{g^4}+\frac{29}{10}a_2^2+\frac{91}{144}\frac{a_3^3}{a_2^2}\right] \tag{14}$$

$$\dot{\theta}_1\left[\frac{\Gamma(1/3)}{54}a_3\left(\frac{6}{a_2}\right)^{4/3}-Pr^{1/3}\right]\frac{1}{ga^2} = Pr^{1/3}+\left[\frac{\Gamma(1/3)}{3}\left(\frac{6}{a_2}\right)^{1/3}-\frac{\Gamma(2/3)}{18Pr^{1/3}}\left(\frac{6}{a_2}\right)^{2/3}\cdot\frac{a_3}{a_2}+\frac{V}{Pr^{2/3}a_2}+\frac{\Gamma(1/3)}{324Pr}\left(\frac{6}{a_2}\right)^{7/3}S\right]\theta_1 \tag{15}$$

where

$$V = \frac{1}{2}\left[\frac{\dot{g}}{g^2}\left(Pr-\frac{3}{10}\right)+\frac{1}{4}\left(\frac{a_3}{a_2}\right)^2-\frac{1}{5g}\frac{\dot{a}_2}{a_2}\right]$$

and

$$S = -\frac{Pr}{2}\frac{\dot{g}}{g^2}a_3-\frac{1}{15}\left(a_2^2-2\frac{\dot{g}}{g^2}-\frac{\ddot{g}}{g^3}\right)+\frac{7}{30}\frac{a_3}{a_2}\left(\frac{3}{2}\frac{\dot{g}}{g^2}a_2+\frac{\dot{a}_2}{g}\right)-\frac{35}{216}\frac{a_3^3}{a_2^3}.$$

Equations (14) and (15) are first order ordinary differential equations which have the initial conditions that $a_2(0)$ and $\theta_1(0)$ are equal to the Hiemenz flow values of wall shear and heat flux.

4. RESULTS AND CONCLUSIONS

Equations (14) and (15) can readily be integrated numerically. Results are presented for two different forms of $g(\tau)$. The wall shear and heat flux results are presented in ratio to their quasi-steady responses denoted by $\tau_{w,QS}$ and $q_{w,QS}$. These ratios reflect the error which would result from using the quasi-steady approximations for problems which are highly unsteady.

Example 1: $g(\tau) = 1+A\sin(\omega\tau)$

This case represents a stagnation point flow for which at time $\tau = 0$ the freestream velocity begins to oscillate in magnitude with amplitude A and frequency ω about a mean velocity. Calculations were carried out for $A = 0{\cdot}1$ and for various frequencies ω. At large τ when starting transients have disappeared, neither the wall shear or heat flux fluctuate with equal amplitude about the quasi-steady solutions. The

of maximum unsteadiness. In addition the wall heat flux ratio fluctuates predominately above the quasi-steady solution. Thus the time mean average wall heat flux over a cycle is enhanced considerably by the increase in amplitude of the freestream velocity fluctuation. It is to be noted

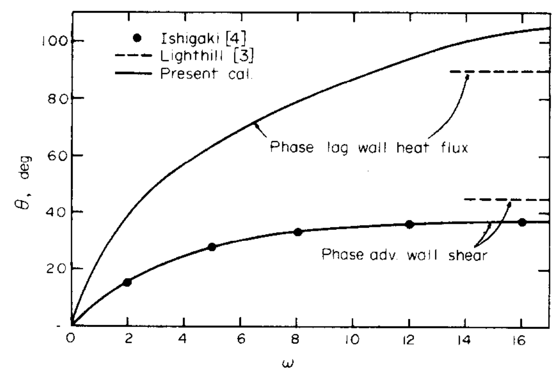

FIG. 1. Phase shifts θ of maximum wall heat flux and shear relative to the maximum freestream velocity for oscillating freestream: $g(\tau) = 1+A\sin(\omega\tau)$, $A = 0{\cdot}1$, $Pr = 0{\cdot}72$.

that the maximum unsteadiness of the heat flux occurs while the boundary layer is experiencing an adverse pressure gradient and the wall shear is approximately at its minimum value. The preceding clearly demonstrates that freestream flow oscillations can greatly enhance surface heat transfer for certain combinations of frequency and amplitude. The phase advance of the wall shear for the large ($A = 0{\cdot}3$) and small amplitude ($A = 0{\cdot}1$) cases were

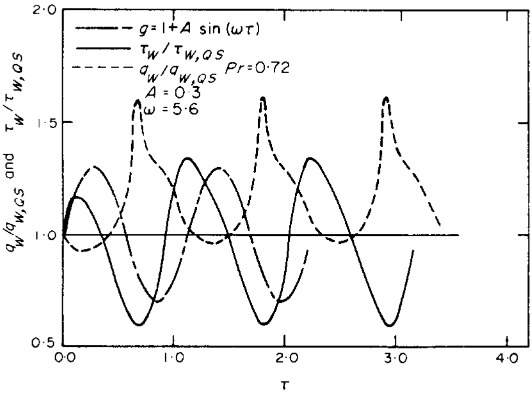

FIG. 2. Variation of wall heat flux and shear stress ratios with time for stagnation point flow: $g(\tau) = 1 + A\sin(\omega\tau)$.

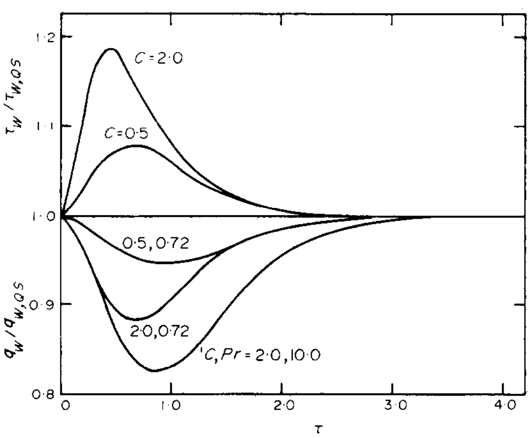

FIG. 3. Variation of wall heat flux and shear stress ratios with time for stagnation point flow: $g(\tau) = 1 + c(1 - e^{-\tau^2})$, $c > 0$.

nearly identical for a given ω. However, the phase lag of the wall heat flux increased from 65° for the small amplitude case to 125° for the large amplitude case at a frequency of $\omega = 5.6$.

The author believes that some cases of enhanced surface heat transfer under oscillatory flow conditions are associated with transient boundary layer separation or incipient separation. However, flow cases involving transient flow separation cannot presently be analyzed due to the limitations imposed by the series inversion used, see [1]. A paper extending the present method of analysis to the study of heat transfer in transient separating boundary layers will be forthcoming.

Example 2: $g(\tau) = 1 + c(1 - e^{-\tau^2})$

This case represents a stagnation point flow in which the magnitude of the freestream velocity increases or decreases exponentially with time to a new steady value for large τ. Figure 3 shows the transient wall shear and heat flux ratios for accelerating flows, $c > 0$. For accelerating flows the wall shear, τw, is greater than the quasi-steady results and the heat flux, qw, is less than the quasi-steady results. Just the opposite behavior is found to occur for decelerating flows. The wall heat flux for both accelerating and decelerating flow exhibits increased unsteadiness with increasing Prandtl number and corresponding larger times required for the thermal transients to disappear and for steady state to be reached. To the author's knowledge heat transfer results have not been reported for this case.

In conclusion, the method of solution presented can also readily be utilized to analyze the thermal response of stagnation point flows for other forms of $g(\tau)$ or for cases where either or both T_w and T_∞ are functions of time.

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